Improved Discretization of Grounding Lines and Calving Fronts using an Embedded-Boundary Approach in BISICLES

D.F. Martin (DFMartin@lbl.gov)[1], H.S. Johansen[1], P.O. Schwartz[1], E.G. Ng[1]

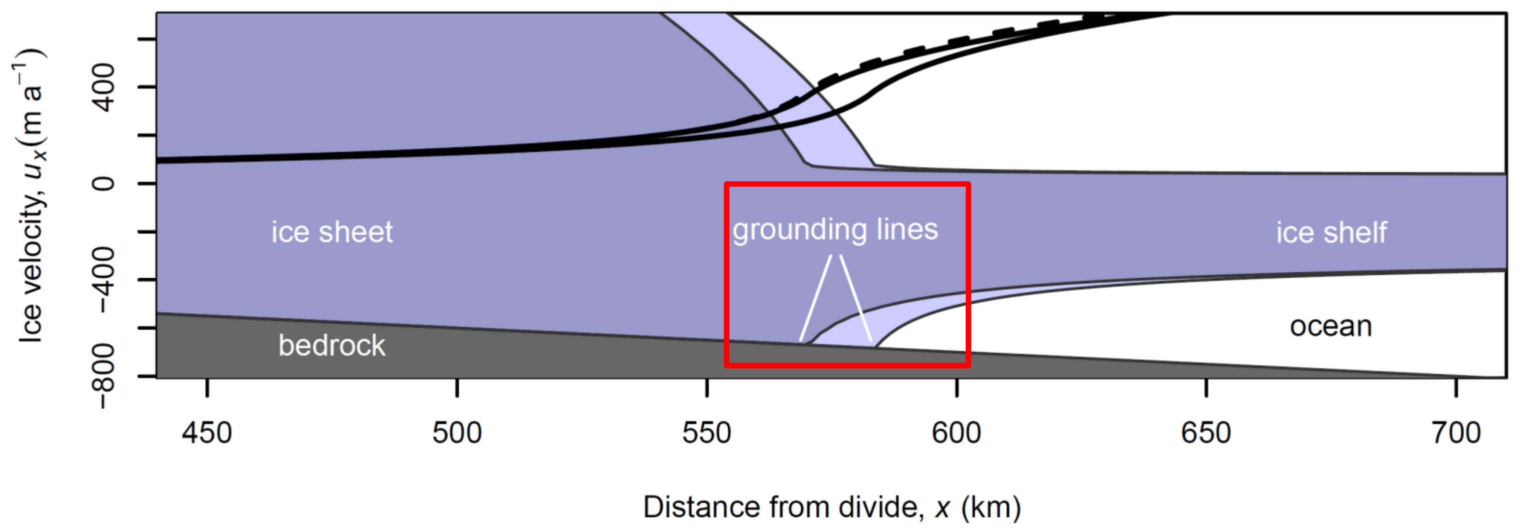
1. Lawrence Berkeley National Laboratory, Berkeley, CA, USA

Motivation

Correctly representing grounding line and calving-front dynamics is of fundamental importance in modeling marine ice sheets, since the configuration of these interfaces exerts a controlling influence on the dynamics of the ice sheet. Traditional ice sheet models have struggled to correctly represent these regions without very high spatial resolution due to the dynamic complexity of the region around the grounding line.

The Grounding line as a Multifluid Interface

- In Marine ice sheets, the grounding line is the location at which ice flowing to the sea thins enough to begin to float.
- The grounding line marks the "phase-change" transition from a grounded ice sheet to a floating ice shelf (introduces a "kink"):



Schematic of two grounding lines – flow is from left to right, black line is the ice velocity. Different rates of ice accumulation or changes in ice-shelf buttressing will drive the grounding line forward or backward.

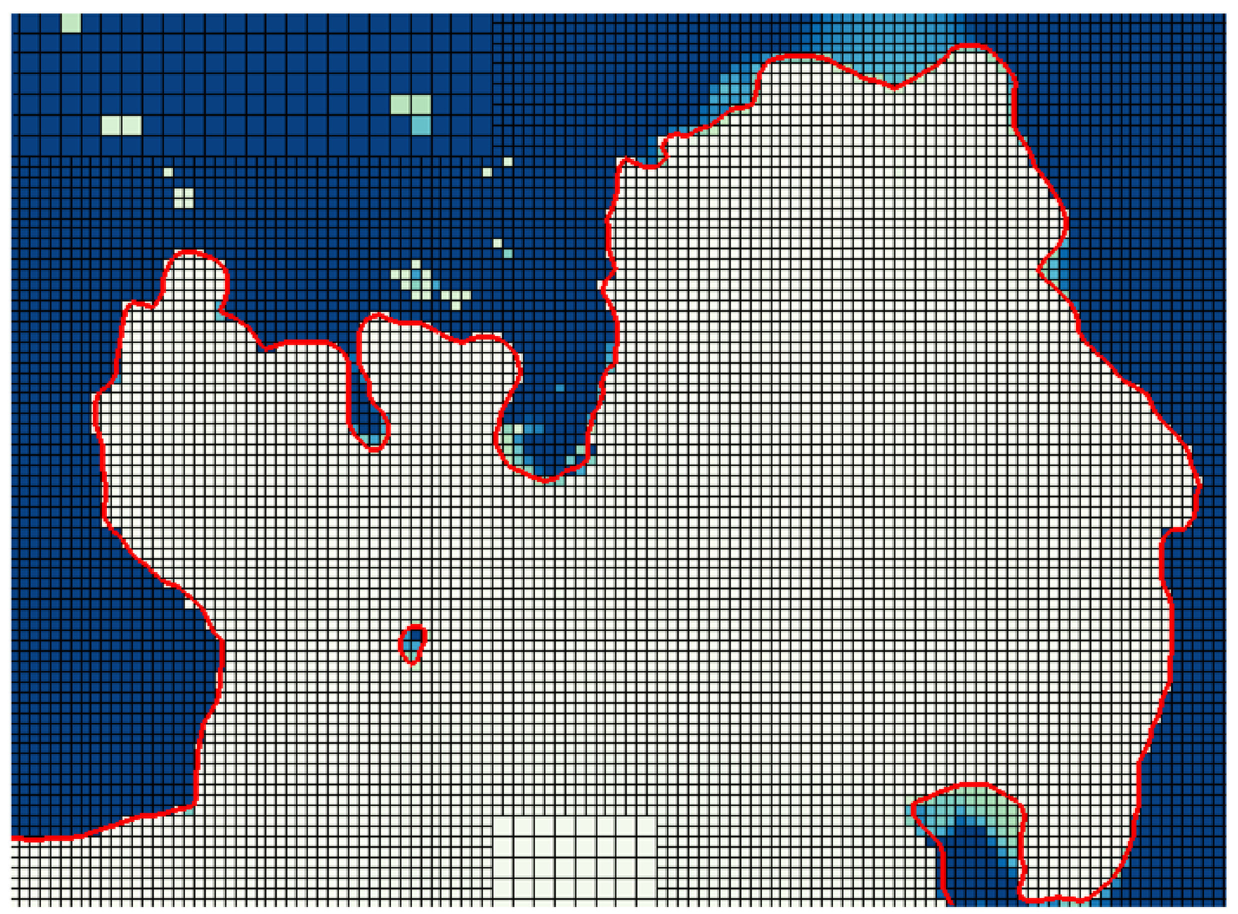
- Recent results have demonstrated that **very fine spatial resolution** (better than 1 km, maybe ~250m) is needed to resolve ice-dynamic features near grounding lines.
  - Computationally prohibitive for uniform-resolution modeling of large ice sheets. (BISICLES uses adaptive mesh refinement)
  - Not model-specific – reported by many authors for many different models.
  - Grounding lines can move ***arbitrarily fast*** depending on bathymetry (z_b), ice thickness (H) → *contact point problem*.
- These point to that fact that in hydrostatic models, the grounding line represents a **singular set**:
  - Basal friction drops to zero (average u equation)
  - Shallow-shelf Approximation (SSA)-type elliptic momentum-balance equations have discontinuous non-linear coefficients:

In 1D: $\left(\beta - \frac{\partial}{\partial x}\left(4\mu H \frac{\partial}{\partial x}\right)\right) u_b = -\rho g H \frac{\partial s}{\partial x}$ (grounded ice – has friction)

$\left(\overset{0}{\cancel{\beta}} - \frac{\partial}{\partial x}\left(4\mu H \frac{\partial}{\partial x}\right)\right) u_b = -\rho g H \frac{\partial s}{\partial x}$ (floating ice – no friction)

Embedded Boundaries for Complex Geometries

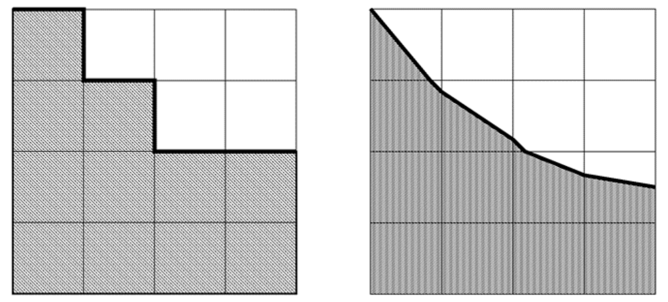
- In BISICLES (and commonly elsewhere), grounding lines and ice margins are forced to cell faces using a "stair-step" discretization, which is inaccurate for grounded ice calculations at low-resolution.



BISICLES Pine Island Glacier calculation with interpolated grounding line (red contours) on a 1km adaptive mesh.

The grid stair-step representation of basal friction (blue), shows lower values in partial cells near the grounding line.

- Chombo's Embedded-boundary (EB) discretization could improve representation of grounding lines, grounded areas, and ice margins using a **cut-cell** approach.



Representing a grounding line using stair-step (left) and embedded-boundary (right) discretizations.

Opportunities & Next steps

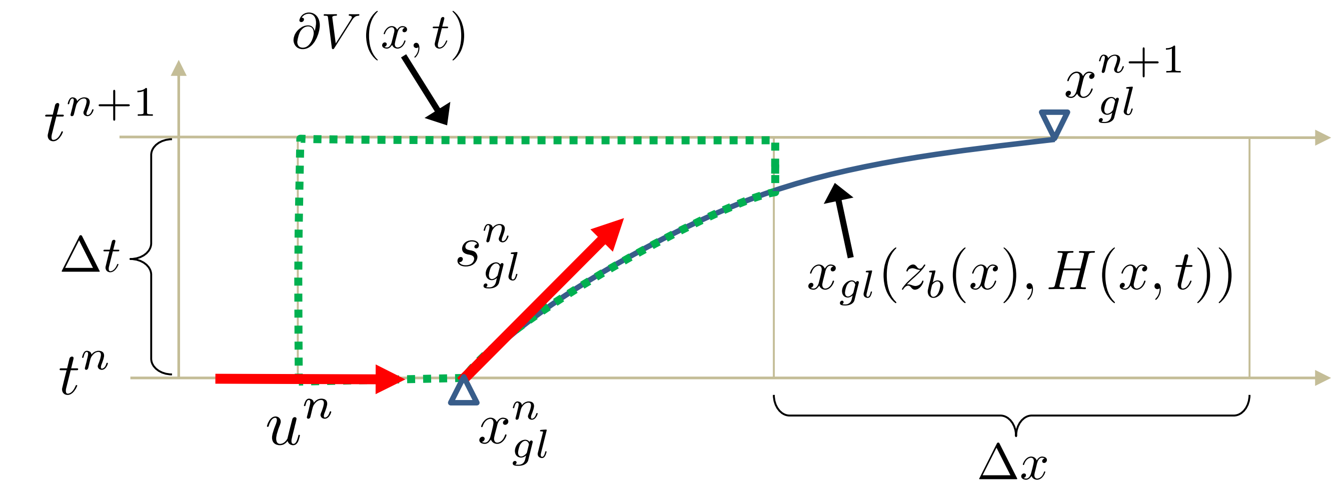
- Extend to 2D formulation to treat grounding line as a "multi-fluid" EB, with jump conditions used in time integration and solvers.
- Track grounding line motion in space-time for more accurate total integral of grounded area, for improved basal friction calculation.
- Investigate error and stability properties, hopefully with the ability to take larger time steps without losing accuracy.

Space-time formulation (1D)

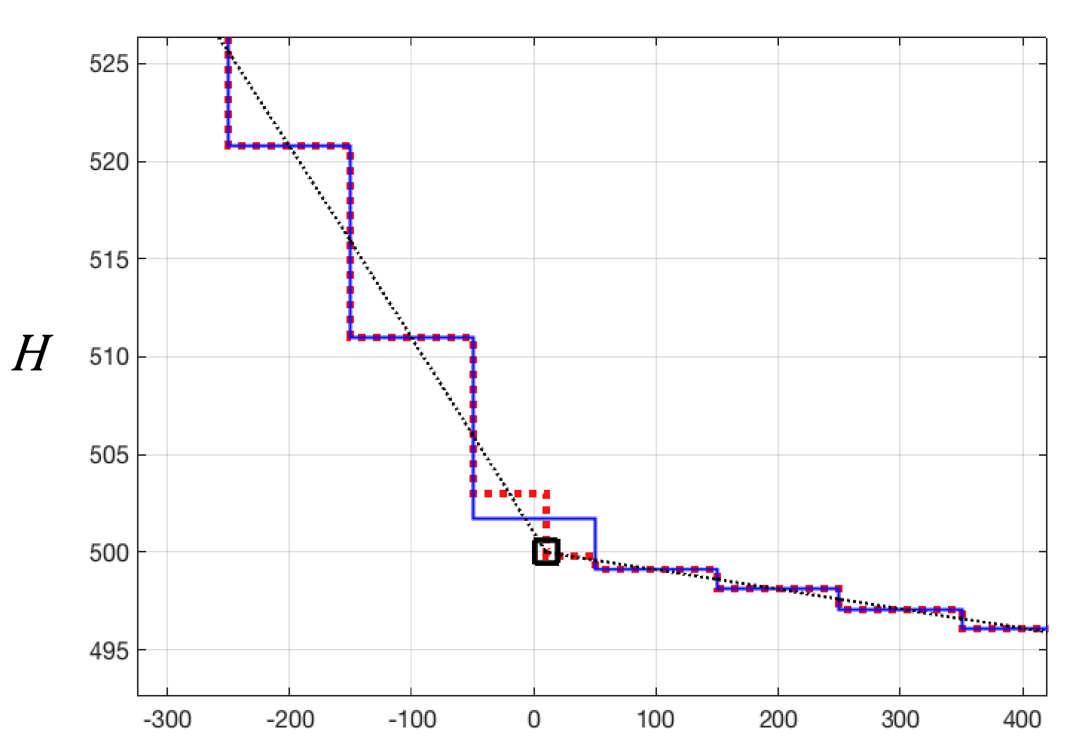
- Grounding line speed in BISICLES can be much faster than ice velocity, due to the interaction of ice thickness and bathymetry:

$$s_{gl} = \frac{\partial x}{\partial t}\bigg|_{\text{buoyancy}=0} = -\rho_i \frac{\partial H}{\partial t} \boxed{\left(\rho_i \frac{\partial H}{\partial x} + \rho_w \frac{\partial z_B}{\partial x}\right)^{-1}}$$

- Consider a two-domain problem with prescribed *jump conditions* across the grounding-line interface x_{gl} (where $[f] = (f_R - f_L)$):
  → Thickness $[H]$ = 0, ice velocity $[u]$ = 0, and stress $[\tau] = [\mu \frac{\partial u}{\partial x}] = 0$
- Grounded, floating phases are discretized over a space-time volume, reconstructing a multivalued solution with jump conditions:



- Grounding line location is calculated using a cubic polynomial fit to cell averaged ice thickness, grounded and floating, enforcing $[H]$ = 0 at x_{gl}:



This shows the conservative reconstruction of ice thickness from cell averages (blue line) to sub-cell polynomial fit (black dotted line). This is used to iteratively find the grounding line (black square). The reconstructed sub-cell averages (red dotted line) match the polynomial fit, the jump condition, and exactly conserve total ice thickness.

- Velocity field solver uses the correct grounded area, then discretizes momentum-balance equation with 4[th]-order accurate EB derivatives.